V. Kehr-Candille
(ONERA)

E-mail: veronique.kehr-candille@onera.fr

DOI: 10.12762/2018.AL14-04

Modelling the Damping at the Junction between Two Substructures by Non-Linear Meta-Models

We are interested in the modelling of the damping at the junction between two substructures. We model the connection by a meta-model, which takes into account both dissipative and non-linear aspects of the connection. We use the Bouc-Wen meta-model. This model is adapted for insertion into a finite-element model. We obtained a non-linear dynamical system, which can be solved in the time domain with a Runge-Kutta algorithm. A software tool corresponding to this method is developed. To decrease calculation costs, we reduce the size of the system by a Craig-Bampton method. We present an application on an academic test-case, and also a comparison with experimental results.

Introduction

In structural dynamics, vibratory levels depend directly on damping. Therefore, it is necessary to have, from the design phase, tools and models that allow the damping to be correctly represented.

The origin of the energy dissipation in the aeronautical structures is double: on the one hand, material intrinsic damping and, on the other hand, dissipation generated by the friction phenomena at the interfaces between the sub-structures. We are interested here in the representation of this second source of dissipation. For the metallic structures, the dissipation generated at the interfaces is the main damping source.

Modelling friction and interface contact has been the subject of numerous studies (for instance, [1, 2, 3]). Most finite-element software contains contact-modelling modules (Nastran, Abaqus, Aster). However, these approaches, used for the calculation of dynamic responses, lead to extremely long calculation times, which become prohibitive for industrial structures. Our objective is to propose a relatively simplified modelling (meta-model) of the junction between two sub-structures, which allows dissipative and non-linear aspects to be correctly represented, without leading to an excessively long time in the calculations of the dynamic response.

Junction model

Single-degree-of-freedom model

Numerous non-linear dissipation models can be found in literature: we are mainly interested in the Bouc-Wen model [4], which allows the modelling of various dissipation phenomena.

For a single-degree-of-freedom system, the differential equations of this model are:

$$\begin{cases} m\ddot{x} + k_{\text{linear}}\, x + z = f_e(t) \\ \dot{z} = \alpha\left[A\dot{x} - \beta|\dot{x}|\,|z|^{n-1} z - \gamma\dot{x}|z|^n \right] \end{cases} \quad (1)$$

The Bouc-Wen model introduces an additional degree of freedom z. The mass of the system is m, the excitation force is denoted by $f_e(t)$. The variable z comprises both non-linear stiffness and non-linear damping aspects.

This model is described by 5 parameters: α, β, γ, A and n (n is not necessarily an integer).

We present Figure 1 to illustrate an example of a response of a single-degree-of-freedom system ($m=1$, $k_{linear}=200$, $f_e(t)=2.\sin(4\pi t)$, $\alpha=200$, $A=1$ and $n=1$, for two sets of parameter values β and γ. For dissipative systems, the most representative curve is the hysteresis curve (hysteresis cycle): displacement x / force z.

Use in a finite-element model

The models presented above are well known. The originality of our approach is in their inclusion in finite-element models.

We consider a Bouc-Wen model inserted between two degrees of freedom of nodes A and B.

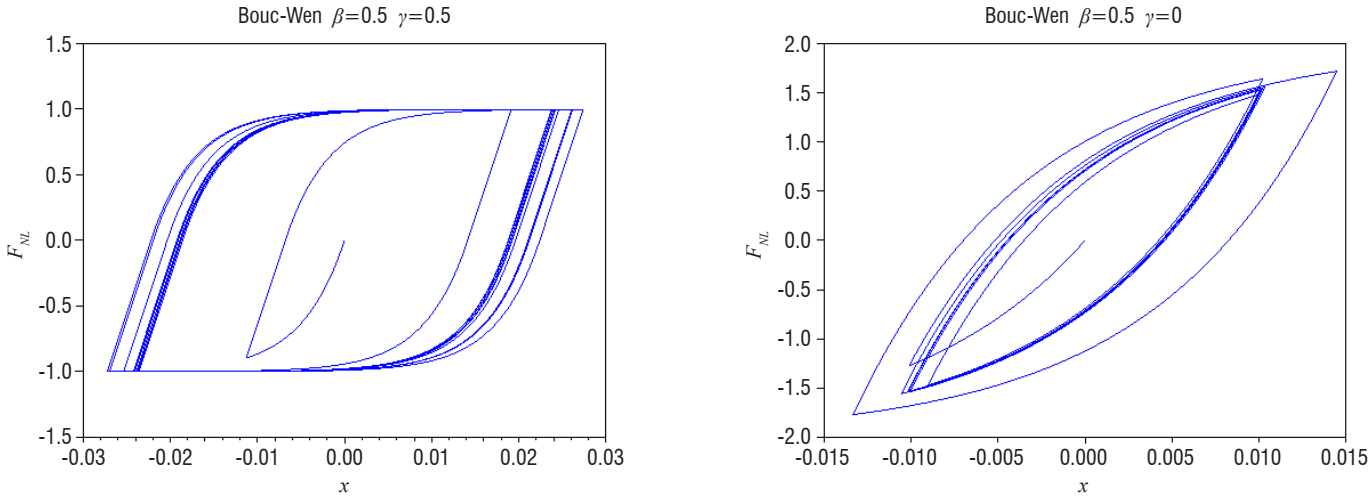

Figure 1 – Hysteresis cycle for a one degree of freedom model

It should be noted that this system is introduced between two degrees of freedom, and not between two nodes. For instance, one can choose to link only the x degree of freedom of node A and the x degree of freedom of node B by a Bouc-Wen model, without linking the y and z degrees of freedom for these two nodes.

Figure 2 – Diagram of the integration of the Bouc-Wen model between two sub-structures

The differential system is written as follows:

$$\begin{cases} \mathrm{M}\ddot{\mathrm{X}} + \mathrm{C}\dot{\mathrm{X}} + \mathrm{KX} + \mathrm{F}_{NL}(z) = \mathrm{F}_e(t) \\ \dot{z} = \alpha\left[A\dot{x} - \beta|\dot{x}|\,|z|^{n-1} z - \gamma\dot{x}|z|^n \right] \end{cases} \tag{2}$$

where

$$\begin{cases} \mathrm{F}_{NL}(i_A) = -z \quad \mathrm{F}_{NL}(i_B) = +z \quad \mathrm{F}_{NL} = 0 \text{ elsewhere} \\ x = x_B - x_A = \mathrm{X}(i_B) - \mathrm{X}(i_A) \\ i_A = n^o \mathrm{ddl}\, x_A \quad i_B = n^o\, \mathrm{ddl}\, x_B \end{cases} \tag{3}$$

The Bouc-Wen model introduces an additional degree of freedom z in the initial system. The non-linear force F_{NL} depends on this additional degree of freedom.

This model can be easily extended to several Bouc-Model models. We introduce as many additional degrees of freedom as Bouc-Wen models.

Resolution

System resolution

The system described above is a time-domain non-linear system, which is well adapted to a resolution by the classical Runge-Kutta method of order 4.

The Runge-Kutta algorithm requires the system to be expressed in the form of a 1st order differential system:

$$\dot{\mathrm{Y}} = f(\mathrm{Y}) \quad \text{avec } \mathrm{Y} = \begin{cases} \mathrm{Y}_1 = \dot{\mathrm{X}} \\ \mathrm{Y}_2 = \mathrm{X} \\ \mathrm{Y}_3 = z \end{cases} \tag{4}$$

The function f is defined by:

$$\begin{cases} \dot{\mathrm{Y}}_1 = -\mathrm{M}^{-1}\mathrm{K}\,\mathrm{Y}_2 - \mathrm{M}^{-1}\mathrm{C}\,\mathrm{Y}_1 - \mathrm{M}^{-1}\mathrm{F}_{NL} + \mathrm{M}^{-1}\mathrm{F}_e(t) \\ \dot{\mathrm{Y}}_2 = \mathrm{Y}_1 \\ \dot{\mathrm{Y}}_3 = \alpha\left[A\dot{x} - \beta|\dot{x}|\,|z|^{n-1} z - \gamma\dot{x}|z|^n \right] \end{cases} \tag{5}$$

where $z = \mathrm{Y}_3 = +\mathrm{F}_{NL}(i_B) = -\mathrm{F}_{NL}(i_A)$ and $\dot{x} = \mathrm{Y}_1(i_B) - \mathrm{Y}_1(i_A)$

For several Bouc-Wen models, the last equation is modified:

$$\dot{\mathrm{Y}}_3(j) = \alpha_j\left[A_j\dot{x}_j - \beta_j|\dot{x}_j|\,|z_j|^{n-1} z_j - \gamma_j\,\dot{x}_j|z_j|^n \right] \tag{6}$$

where $\mathrm{Y}_3(j) = z_j = +\mathrm{F}_{NL}(i_{Bj}) = -\mathrm{F}_{NL}(i_{Aj})$ and $\dot{x}_j = \mathrm{Y}_1(i_{Bj}) - \mathrm{Y}_1(i_{Aj})$, for j varying from 1 to the number of Bouc-Wen models.

We use the Nastran ® software. We have developed in the internal language of Nastran ® (DMAP) a module that allows several Bouc-Wen models to be inserted between some degrees of freedom of the system (chosen by the user). We also developed in DMAP language the Runge-Kutta algorithm. This module has been implemented in SOL 109 (direct linear transient method).

Reduction method

The previous numerical resolution is directly performed from the initial finite-element system (physical degrees of freedom). This can lead to a prohibitive calculation time for industrial applications.

For linear problems, several model-reduction methods are available to reduce the calculation time. The problem is different for non-linear systems, and it is necessary to develop specific methods (see [5]).

However, in our case, we are dealing with a non-linear system where non-linearities are spatially localized. Therefore, we choose a classical method for a linear system: the Craig-Bampton method.

We briefly recall the different stages of this method:

- the degrees of freedom of the finite-element model are separated into two groups: internal degrees of freedom and boundary degrees of freedom,
- the eigenmodes with clamped boundary degrees of freedom are calculated: modes Φ_m,
- constrained modes are calculated: static response to a unitary imposed displacement of one boundary degree of freedom, with the other boundary degrees of freedom clamped: modes Φ_l,
- the initial finite-element system is projected on the base $\Phi=[\Phi_m\ \Phi_l]$.

The degrees of freedom involved in the Bouc-Wen models are considered as boundary degrees of freedom: the value of these degrees of freedom can be accessed directly during the non-linear transient resolution.

The second software version, which uses the Craig-Bampton reduction method, includes two stages:

- in the first stage, we use Nastran to calculate clamped eigenmodes Φ_m and constrained modes Φ_l, and to carry out the projection on the basis $\Phi=[\Phi_m\ \Phi_l]$,
- in a second stage, we carry out the time domain calculation, on the reduced system, with a Runge-Kutta algorithm. This second phase is computed in Fortran (this stage is independent from the Nastran software).

Academic application

Computing transient response

The above method is applied to a simple academic system, including masses and springs (see Figure 3). This model is the assembly of two sub-structures, and each sub-structure includes 4 masses ($m=10$kg) and 3 springs ($k=10^5$N/m). substructure 1 is clamped at one end. The excitation force is applied to substructure 2 at the other end: $F_e(t)=\cos(2\pi f_0 t)$, where $f_0=2$Hz. A Bouc-Wen model (without linear elasticity $k_{\text{linear}}=0$) links the two substructures.

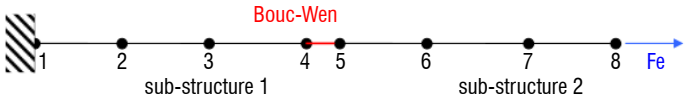

Figure 3 – Mass/spring system

First, in order to validate the software, we consider a non-dissipative and linear case: if we take the values of the Bouc-Wen model parameters $\beta=\gamma=0$, $A=0$, $n=1$, $\alpha=10^5$, the junction is equivalent to a simple spring (non-dissipative and linear), whose stiffness is $k=\alpha A$. We can check that the result is the same as that obtained for a classical linear case, replacing the Bouc-Wen model by a spring.

Then, we calculate the transient response of the system for various parameter values. All of the calculations are performed on the time interval $0-5s$, with a constant time step $\Delta t=10^{-3}s$.

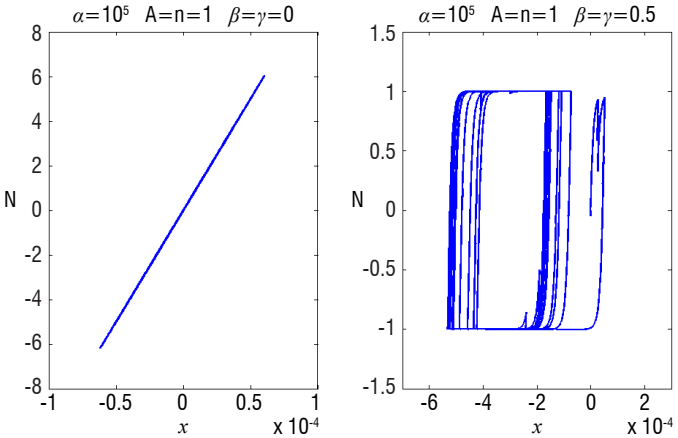

Figure 4 – Hysteresis cycle for the mass/spring system

Craig-Bampton reduction method

We verify, on the academic model, the Craig-Bampton reduction method. Indeed, the validity and efficiency of this method are well known for linear systems. The use of the Bouc-Wen model leads to a non-linear system. However, the non-linearities are localized and only concern the boundary degrees of freedom; therefore, clamped eigenmodes and constrained modes are calculated for the linear part.

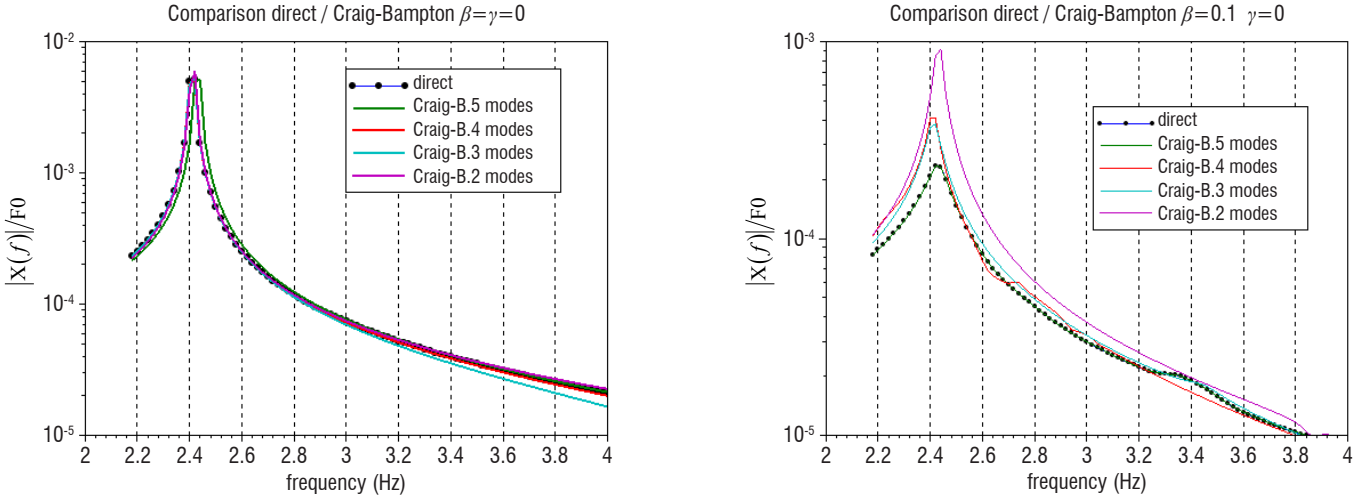

Figure 5 – Comparison of direct responses and responses with the Craig-Bampton method for a linear case (on the left) and a non-linear case (on the right)

We use the transient excitation force $f_e(t) = F_0 \dfrac{\sin(2\pi f_0 t)}{t}$ with $f_0 = 20$ Hz, which allows the two first eigenmodes (2.4 Hz and 7.2 Hz) to be caught. In Figure 5, we compare the direct response (reference response) and the response obtained by the Craig-Bampton method, with various numbers of clamped modes.

We note that convergence is reached for this non-linear system, but for a higher number of modes than for linear systems.

Thus, we can consider that the use of the Craig-Bampton method is valid for non-linear systems with localized non-linearities. However, without general mathematical results, a convergence study is essential.

Comparison with experiment

Description of the mock-up

Within the framework of the MAIAS project, a mock-up, representative of an aeronautic structure, has been built. It is composed of a long part and a short part, linked by a bracket junction (see Figure 6). A detailed description of the mock-up can be found in [7]. We are especially interested in the modelling of this junction.

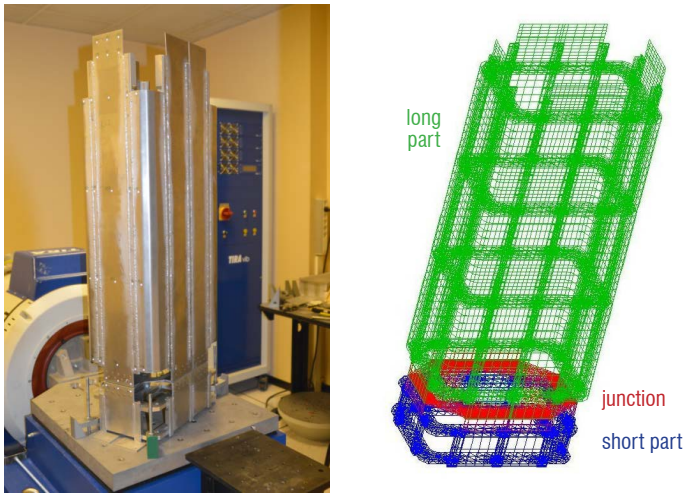

Figure 6 – Mock-up: experimental test (left) and finite-element mesh (right)

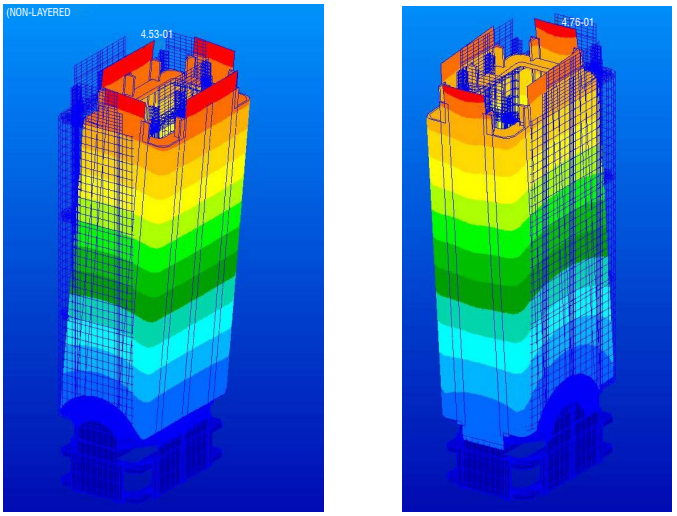

Figure 7 – Eigenmodes (on the left Mode 1: 37.9 Hz simulation, 35 Hz experiment; on the right Mode 2: 72.7 Hz simulation, 69 Hz experiment)

This mock-up has been used for experiments (see [7] and [8]). The experiments performed in [7] allowed he initial finite-element model to be improved, comparing experimental and computed eigenmodes.

We are interested in the experiment performed in [8] (see Figure 6): acceleration is applied at the base of the mock-up, and the response is measured at various points, for several excitation levels. In Reference [8], experimental curves clearly show a non-linear behavior (the frequency response functions depend on the excitation level).

Comparison: experimental and numerical results

In Reference [7], a detailed representation of the bracket junction has been used. Our purpose is to replace this refined junction model by a Bouc-Wen meta-model, and to compare the results thus obtained with experimental results [8].

We considered the finite-element model of [7], and then we removed the finite-element part corresponding to the junction and replaced it by springs and Bouc-Wen models. We introduced 8 springs on each small face, and 13 springs on each large face (42 springs in total), in the three directions x, y and z. At the center of each small face and

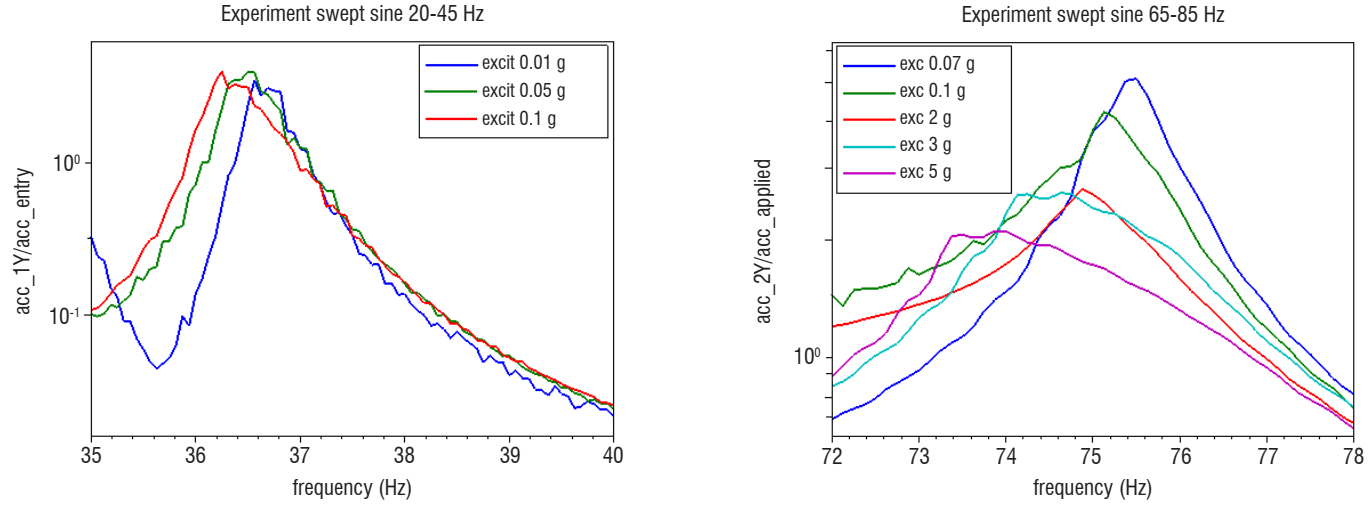

Figure 8 – Experimental responses of the mock-up (in the neighborhood of resonance peaks)

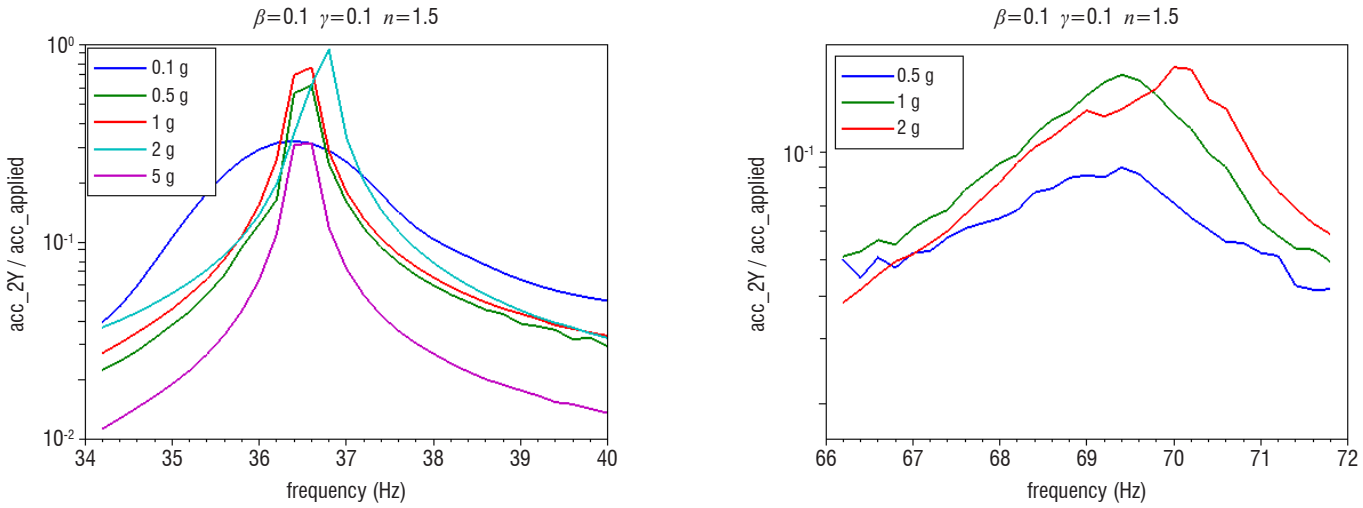

Figure 9 – Simulated responses of the mock-up

each large face, we introduced a mixed junction: a Bouc-Wen model in the vertical direction z, and springs in the two other directions.

In a first stage, the spring stiffness values were modified, in order to use the same values as in the experiment for the first two eigenmodes (see Figure 7).

In a second stage, we included in the finite-element system the 4 Bouc-Wen models, and we performed transient dynamic response calculations, using the Craig-Bampton method. Simulations were performed for several levels of the acceleration applied.

We present below the frequency response curves: experimental (Figure 8) and simulated (Figure 9). These curves are shown separately, because they do not correspond exactly to the same excitations. In fact, experimental excitations are swept sines; the numerical simulation of swept sines leads to very high calculation times. Moreover, numerical problems (divergence of the numerical scheme) occur after a certain period. Thus, for the numerical simulations, we have used the time excitation $A_0 \dfrac{\sin(2\pi f_0 t)}{t}$ with $f_0 = 200\,\text{Hz}$, which allows the first bending modes to be excited, for reasonable time calculations.

Comments

At first sight, the comparison between the simulation and experimental results is not very good. First, one can note significant differences for resonance frequencies. Moreover, on experimental curves, the resonance frequency decreases and the damping increases when the excitation amplitude increases, while the opposite seems to occur on simulation curves (however, the trend is not obvious).

With regard to the resonance frequency, as a first step, the conservative finite-element model was adapted to fit the experimental eigenvalues. Then, the introduction of damping and non-linear aspects through the Bouc-Wen model leads to a discrepancy between the initial conservative eigenfrequencies and the resonance peak frequencies.

For the second point, for the Bouc-Wen model, the evolution of a resonance frequency and damping rate with the excitation amplitude is not very clear (this fact has been checked in many other examples).

However, despite these differences between experiment and simulation, this test case shows that the Bouc-Wen model allows both the dissipative and the non-linear aspects of the junction to be represented, with a much simpler model of the junction, and lower computation time.

Of course, the approach has to be improved. In particular, a rigorous method to identify the Bouc-Wen parameters must be developed. Moreover, this experimental mock-up was certainly too complicated for a first application.

Conclusion

We have presented a method to model the junctions in structural dynamics with meta-models. This approach allows both the dissipative aspect and the non-linear aspect of the junction to be taken into account.

This approach could be completed by the development of a method for identifying the values of the meta-model parameters *a priori*. Methods to identify Bouc-Wen parameters from experimental hysteric loops are under investigation.

The resolution method itself could be improved, on the one hand by improving the time domain non-linear algorithm and, on the other hand, by using the latest research developments concerning reduction methods in non-linear cases ■

Acknowledgements

This work was carried out within the framework of the collaborative project MAIAS (*Maîtrise de l'Amortissement Induit dans les ASsemblages*, *i.e.*, induced damping control in assemblies), under FUI financing (French government).

References

[1] S. BOGRAD, P. REUSS, A. SCHMIDT, L. GAUL - *Modeling the Dynamics of Mechanical Joints*. Mechanical Systems and Signal Processing 25, 2801–2826 (2011).

[2] H. FESTJENS - *Contribution à la caractérisation et à la modélisation du comportement dynamique des structures assemblées*. LISMMA Supmeca doctoral thesis (2014).

[3] A. CAIGNOT - *Prédiction par essais virtuels de l'amortissement dans les structures spatiales*. LMT ENS Cachan doctoral thesis (2009).

[4] M. ISMAIL, F. IKHOUANE, J. RODELLAR - *The Hysteresis Bouc-Wen Model, a Survey*. Archives of Computational Methods in Engineering 16,161-188 (2009).

[5] F. A. LÜLF, D.M. TRAN, R. OHAYON - *Reduced Bases for Nonlinear Structural Dynamic Systems: a Comparative Study*. Journal of Sound and Vibration 332, 3897-3921 (2013).

[6] C. HAMMAMI, E. BALMES, M. GUSKOV - *Conception et validation d'une liaison boulonnée dissipative*. 11ème Colloque National en Calcul des Structures (Giens), 13-17 May 2013.

[7] C. HAMMAMI - *Intégration de modèles de jonctions dissipatives dans la conception vibratoire de structures amorties*. ENSAM doctoral thesis (2014).

[8] A. SÉNÉCHAL - *Présentation du projet MAIAS*. colloque Aero'Nov Connection, 28 and 29 January 2014.

AUTHOR

Véronique Kehr-Candille graduated from ECL Ecole Centrale de Lyon in 1987. She joined ONERA in 1988, first in the Structure Department, and then in the Structural Dynamics Department. She carried out scientific studies in various fields of structural dynamic simulation: vibroacoustic, acoustic materials, viscoelascity and rotor dynamics. Her current main research interests are damping models and nonlinear structural dynamic response.